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Numerical study of interaction of vortex structures in plasmas and fluids[1]

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The results of numerical study of evolution and interaction of the vortex structures in the continuum, and, specifically, in plasmas and fluids in the 2D approach, when the Euler-type equations are valid, are presented. The set of the model equations $e_i d_t x_i = \partial_{y_i} \mathrm{H}/B$, $e_i d_t y_i = -\partial_{x_i} \mathrm{H}/B$, $\partial_t \rho + \mathbf{v}\cdot\nabla\rho = 0$, $\mathbf{v} = -(\hat{\mathbf{z}}\times\nabla\psi)/B$, $\Delta\psi = -\rho$ *describing the a continuum or quasi-particles with Coulomb interaction models, where* ρ *is a vorticity or charge density and* ψ *is a stream function or potential for inviscid fluid and guiding-centre plasma, respectively, and* H *is a Hamiltonian, was considered. For numerical simulation the CD method specially modified was used. In terms of vortex motion of fluids the results of numerical experiments, specifically, showed that for some conditions the interaction of vortexes in continuum may be nontrivial and, as for the "classic" FAVRs, lead to formation of complex forms of vorticity regions, for example, the vorticity filaments and sheets, and also can ended to formation of the turbulent field. The undertaken approach may be effective in studying of the atmospheric and Alfvén vortex dynamics, and also useful for the interpretation of effects associated with turbulent processes in fluids and plasmas.*

1. Basic equations

In this paper we study numerically the evolution and interaction of the vortex structures (so-called FAVRs [1]) in the continuum, and, specifically, in plasmas and fluids in 2D approach, when the Euler-type equations are valid. In general case the set of the model equations describing the a continuum (inviscid incompressible fluid) or quasi-particles (charged filaments aligned with a uniform field **B**) with Coulomb interaction models is the following:

$$e_i d_t x_i = \partial_{y_i} \mathrm{H}/B, \quad e_i d_t y_i = -\partial_{x_i} \mathrm{H}/B, \quad \partial_\nu \equiv \partial/\partial\nu,$$
$$\partial_t \rho + \mathbf{v}\cdot\nabla\rho = 0, \quad \mathbf{v} = -(\hat{\mathbf{z}}\times\nabla\psi)/B, \tag{1}$$
$$\Delta\psi - f = -\rho$$

where e_i is the charge per unit length of the filaments or the strength (circulation) of discrete vortex, ϕ is a z-component of vorticity ζ or charge density ρ, and ψ is a stream function or potential for 2D flow of inviscid fluid and guiding-centre plasma, respectively, and H is a Hamiltonian. Note, that in the continuum (fluid) model B=1 in the Hamiltonian eqs. (1). Function f=0 for the continuum or quasi-particles (filaments) with Coulomb interaction models [2], and $f = k^2\psi$ for a screened Coulomb interaction model [3]. We will consider here only a case f=0, and generalization of our approach for $f = k^2\psi$ is rather trivially.

For numerical simulation the contour dynamics (CD) method [1], to some extent modified, was used. This method gives a possibility not only to observe evolution of single contour, but also to study the interaction between vortexes having different symmetry order (different modes). Let us demonstrate it in the next paragraph.

2. Numerical results and discussion

Let us consider some results of numerical simulation in terms of the vortex motion of the inviscid incompressible fluid, as more visual. In general, to study the evolution of vortex structures with different symmetry orders it is necessary to insert a small amplitude disturbance $r = R_0[1+\varepsilon\cos(m\alpha - \tilde{\omega}_m t)]$ (where R_0 is a conditional radius, ε is an eccentricity, m is symmetry order (mode), α is an angle and $\tilde{\omega}_m = \zeta_0(m-1)/2$) to the constant vorticity circle region. But, accounting that the results of evolution for one and two vortices with different m were described in detail in [4], let us stay on results on interaction of vortices and consider the most simple case of circle vortices when m=1 and, therefore, $\tilde{\omega}_m = 0$. For two vortices the result of the interaction depends on sign of vorticity ζ ("polarity") and the distance δ between boundaries of vortices. For two vortices having opposite polarities for initial distance $\delta = d$ where d is a vortex diameter we observed that the vortices, rotating in opposite directions, move in the same direction and, practically don't interact independently on value of δ. For the vortices having opposite polarity the result of evolution depends essentially on δ. So, for rather small δ the vortices, on a level with rotation about their own axes and around of their common center, interact forming a common vortex region which consists of the vorticities of more small scales. For rather big δ the vortices on a level with rotation about their own axes rotate around of common center, at this, their interaction is reduced to a cyclic change of their shape (so-called "quasi-return" phenomenon [1] is observed). In our numerical experiments we have found that critical initial distance dividing these two types of interaction $\delta_{cr} = 3d/4$.

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To study the interaction of the linearly disposed vortices in more details we have consider the problem for four vortices being at initial time along one line. The simulation of the interaction at linear disposition of four vortices one can observe that for rather big and equal initial distance between vortices the evolution leads to formation of two vorticity regions as a result of more strong interaction of the "outer" vortices with the "inner" ones. At this, the interaction of forming pairs is similar to that of two vortices case. In case $\delta_i = d/2$ we observed the formation of a complex vortex structure which consists of many vorticities of more small scales. The further evolution of this structure leads to formation of complex turbulent field. Let us note that in last case we can also see that the interaction between outer vortices is more strong. This fact can be explained by the fact of more strong "attraction" of outer vortices to the "center of mass" of the vortex system because the outer vortex is attracted to the center by three other vortices, and the inner vortex is attracted to the center by two vortices and, in opposite side, by one outer vortex. To test this statement in the next series of numerical experiments we have arranged outer and inner vortices on different initial distances. As a result, we observed the formation of vortex structure from two inner vortices.

In the next series of experiments we studied the interaction between the vortices disposed at initial time in the corners of appropriate equilateral figures. At this, the following results were obtained. In case of evolution of three vortices with opposite signs of ζ being at initial time in the corners of triangle, we have obtained that a pair of them, having opposite polarities, behaves as well as pair of vortices with opposite polarities in two vortices case, and third vortex does not participate in interaction almost, practically independently on value of δ_i $(i=1,2,3)$. The similar character of interaction is observed for four vortices with opposite signs of ζ being at $t=0$ in the corners of square.

The character of interaction of three and four vortices having the same polarities depends essentially on the distances between them like in the two vortices case. For example, for $\delta = d/2 < \delta_{cr}$ we observed that three vortices are rotated forming one big vortex which consists of many vorticities of more small scales. Similar picture is observed for four vortices being at $t=0$ in the corners of square on distances smaller critical one from another.

3. Conclusion

So, we have presented, as more visual, some results of numerical simulation of eqs. (1) in terms of the vortex motion of the inviscid incompressible fluid. But, as we noted in the beginning, the set of eqs. (1) with $f=0$ may describe also the quasi-particles with Coulomb interaction model. At this, the results presented above can be easily extended on the 2D simple system where the plasma is represented by charged filaments, aligned with a uniform field **B**, that move with the guiding-centre velocity $\mathbf{E}\times\mathbf{B}/B^2$. Moreover, the approach undertaken can be useful and also for other 2D continuum models when $f \neq 0$ in the Poisson eq. (1). They can describe the vortices or filaments with the non-Coulomb interaction. In the last case it is assumed that ions move with the guiding-centre velocity but electrons have a Boltzman distribution, at this, the additional term $f = k^2\psi$ will describe the Debye screening [3]. Another plasma model that can be investigated using the described approach is the Hasegawa-Mima model [5], its equations can also be put in form (1) by way of inclusion of ion polarization drift through the ion equation of motion [3] $d_t\mathbf{v} = (e/M)(-\nabla\varphi + \mathbf{v}\times\mathbf{B})$. As to a hydrodynamic fluid model corresponding to a screened interaction that such model having form (1) has been proposed in [6] to describe the Earth's atmosphere with the equation of motion for the horizontal atmospheric flow $d_t\mathbf{v} = -g\nabla h + R\mathbf{v}\times\hat{\mathbf{z}})$ where h is the atmospheric depth and R is the Coriolis force. Another possible application can take a place in the study of the problems associated with the dynamics of the Alfven vortices in the ionospheric and magnetospheric plasma [7].

In conclusion, let us note that the applications of the dynamics of different types' vortex structures will not rest the problems of study of the processes discussed above. in the power units. Such investigations are now one of the most perspective trends in some technical fields, for example in the problems of study of the vortex motions in working chambers of the varied types' power units, where the working substance may be water, vapor-gas mixtures and plasma [8]. At this, the most actual problem is the investigation of the vortex motions in the spatial regions where there are the flows with velocity shear (for example, near the walls of the boiler of thermal power units and the chambers of the gas turbines), and also the study of the vortex motions in magnetized plasma without dissipation (controlled thermonuclear fusion systems) where the electric currents can lead to formation of very complicated forms of the vortex regions.

References

[1] N.J.Zabusky, M.H.Hughes, K.V.Roberts, *J. Comput. Phys.*, **135** (1997) 220.
[2] J.B.Tailor, B.McNamara, *Phys. Fluids* **14** (1971) 1492.
[3] J.B.Tailor, *Plasma Phys. and Contr. Fusion*, **39** (1997) A1.
[4] V.Yu.Belashov, R.M.Singatulin, *Proc. RNSPE-3*, Kazan, Russia, **2** (2001) 61.
[5] A.Hasegawa, K.Mima, *Phys. Fluids*, **21** (1978) 87.
[6] J.G.Charney, *Geophs. Public. Kosjones Nors.* Videnshap-Acad. Oslo, **17** (1948) 3.
[7] O.A.Pokhotelov, L.Stenflo, P.K.Shukla, *Plasma Physics Reports*, **22** (1996) 941.
[8] V.Yu.Belashov, R.M.Singatulin, *Izv.Vuzov - Pow. Eng. Probl.*, **9** (2001) 103.